ARTICLE NO. 78-1342R

Maximum Likelihood Estimation of Translational Acceleration Derivatives from Flight Data

Richard E. Maine* and Kenneth W. Iliff†
NASA Dryden Flight Research Center, Edwards, Calif.

This paper shows that translational acceleration derivatives, such as pitching moment due to rate of change of angle of attack ($C_{m_{\dot\alpha}}$), can be estimated from flight data with the use of appropriately designed maneuvers. No new development of estimation methodology is necessary to analyze these maneuvers. Flight data from a T-37B airplane were used to verify that $C_{m_{\dot\alpha}}$ could be estimated from rolling maneuvers.

Nomenclature

A,B,C,D,R	= system matrices
AR	= aspect ratio
a_n,a_x,a_y	= normal, longitudinal, and lateral accelerations
b	= reference span
C_A,C_L,C_N,C_Y	= coefficients of axial, lift, normal, and lateral force
C_l,C_m,C_n	= coefficients of rolling, pitching, and yawing moment
c	= mean aerodynamic chord of wing
G	= measurement noise spectral density matrix
g	= acceleration due to gravity
I_X,I_{XZ},I_Y,I_Z	= moments of inertia
J	= cost functional
l	= length from aerodynamic center of wing to aerodynamic center of tail
p	= roll rate
q	= pitch rate
$\bar{q}$	= dynamic pressure
r	= yaw rate
s	= wing area
T	= total time
t	= time
u	= control vector
V	= velocity
v	= bias vector
X,Y,Z	= longitudinal, lateral, and normal axes
x	= state vector
y_ξ	= computed observation vector
z	= measured observation vector
α	= angle of attack
α_t	= tail angle of attack
α_w	= wing angle of attack
β	= angle of sideslip
$\delta_a,\delta_e,\delta_r$	= aileron, elevator, and rudder deflections
ϵ	= downwash angle
η	= measurement noise vector
θ	= pitch attitude
λ	= taper ratio
ξ	= vector of unknowns
ρ	= atmospheric density
φ	= roll attitude

Subscripts

$q,\alpha,\alpha_t,\dot\alpha,\dot\beta,\delta_e$	= derivative with respect to indicated quantity
0	= bias
E	= equivalent

Superscript

*	= matrix transpose

Presented as Paper 78-1342 at the AIAA Atmospheric Flight Mechanics Conference, Palo Alto, Calif., Aug. 7-9, 1978; submitted Aug. 15, 1978; revision received Feb. 12, 1979.

Index categories: Testing, Flight and Ground; Handling Qualities, Stability and Control; Nonsteady Aerodynamics.

*Aerospace Engineer. Member AIAA.
†Aerospace Engineer. Associate Fellow AIAA.

Introduction

ESTIMATION of stability and control derivatives from aircraft flight data is a well-established technology. Translational acceleration derivatives, however, are not currently estimated from flight data. Analytical and wind-tunnel techniques are available to predict translational acceleration derivatives,[1-7] but all previous attempts to estimate such derivatives from flight data have failed.[8-10] This paper shows that translational acceleration derivatives can successfully be estimated from flight data, with the use of appropriately designed maneuvers. Maximum likelihood estimates from flight tests of a T-37B airplane are presented.

Translational Acceleration Derivatives

Translational acceleration derivatives account for the aerodynamic effects of aircraft acceleration along any of the three body axes. When the equations of motion are written in the form given in the Appendix, the translational acceleration derivatives due to a_n, a_y, and a_x become derivatives with respect to $\dot\alpha$, $\dot\beta$, and $\dot V$. This paper concentrates on the derivative $C_{m_{\dot\alpha}}$, pitching moment due to vertical acceleration; the principles developed, however, apply to the estimation of all the translational acceleration derivatives.

Translational acceleration derivatives are derived from approximations of unsteady aerodynamic effects. Exact solutions of unsteady of aerodynamic equations are extremely difficult,[11] but first-order approximations are adequate in the frequency range involved in most aircraft stability and control analysis.

For a better understanding of the source of the derivative $C_{m_{\dot\alpha}}$, it is instructive to consider a simplified version of the unsteady aerodynamics of a straight-wing aircraft of conventional configuration. Pure pitching effects are ignored here, and will be discussed later. Figure 1 shows a two-dimensional side view of such an aircraft, consisting of a wing and a horizontal tail; fuselage, indicial lag, and other effects are ignored. Figure 1a depicts the steady initial flow over the

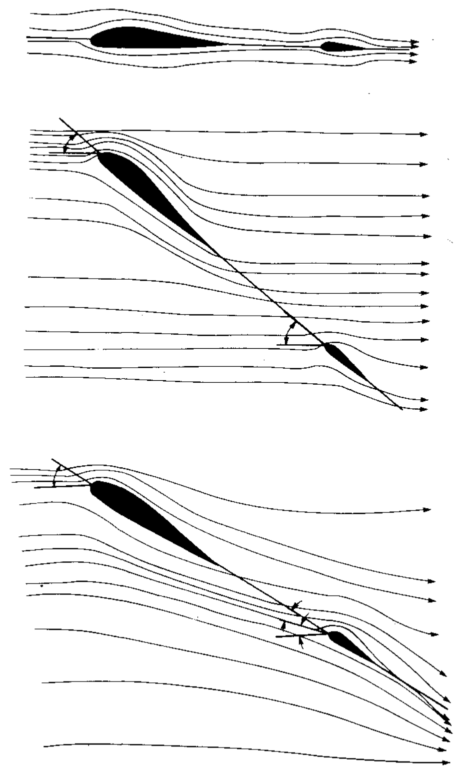

Fig. 1 Simplified phenomenology of $C_{m_{\dot{\alpha}}}$. Angle of attack is exaggerated for clarity. a) Steady initial flow, $\alpha = 0$ deg, b) flow immediately after a rapid step change in α; and c) final stabilized flow.

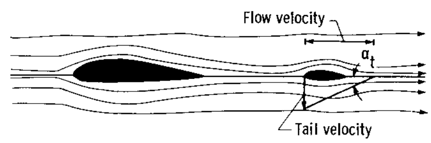

Fig. 2 Simplified phenomenology of C_{m_q}.

wing and tail at 0 deg angle of attack. In Fig. 1b, the flow is shown immediately after a rapid step change in angle of attack, and Fig. 1c shows the final stabilized flow at the new angle of attack. The most significant factor in the pitching moment of the aircraft is α_t, the angle of attack of the tail. In Fig. 1b, the tail angle of attack is equal to α_w, the wing angle of attack. The downwash from the wing has just begun to develop, but it has not reached the tail. In Fig. 1c, the steady-state tail angle of attack equals the wing angle of attack minus the downwash angle, ϵ. If only steady-state aerodynamics are considered, the flow changes immediately from that in Fig. 1a to that in Fig. 1c; however, the nonsteady aerodynamics during the transition period result in a tail angle of attack during that period that is considerably higher than the final stabilized value. The transition period is essentially the time required for the wing downwash to reach the tail. The derivative $C_{m_{\dot{\alpha}}}$ provides a first-order model of the effects of this downwash lag on the pitching moment.

The derivative $C_{m_{\dot{\alpha}}}$ is usually ignored in the stability and control analysis of aircraft flight data, and the downwash lag is considered as an additional component of C_{m_q}, pitching moment due to pitch rate. Figure 2 shows the simplified phenomenology of C_{m_q}. The tail component of C_{m_q} derives solely from the tail angle of attack generated by tail motion. Unsteady aerodynamics and wing downwash are not factors in the derivative C_{m_q}. Thus C_{m_q} and $C_{m_{\dot{\alpha}}}$, although closely related in the mathematical model, are associated with unrelated aerodynamic phenomena.

Since translational acceleration derivatives have generally been ignored in previous flight data analyses, it is pertinent to question the need for estimating them now. There are three reasons why the ability to estimate translational acceleration derivatives from flight data would be useful. First, this capability would assist in validating prediction techniques for translational acceleration and rotary derivatives. At present, comparisons can be based only on combined translational acceleration and rotary effects, which complicates the problem of tracing the source of discrepancies.

Second, comparison of static moment derivatives with predictions is impaired if translational acceleration derivatives are ignored. The equivalent derivatives, those estimated when the translational acceleration derivatives are not modeled, model most aircraft motions adequately. However, correction terms involving the translational acceleration derivatives are necessary in order to accurately compare the predicted and flight-estimated static moment derivatives.[12] Longitudinally, the only important correction term is:

$$C_{m_\alpha} = (C_{m_\alpha})_E + \frac{\rho s c}{4m} C_{L_\alpha} C_{m_{\dot{\alpha}}} \tag{1}$$

The third reason for obtaining translational acceleration derivatives from flight data involves unusual maneuvers. The equivalent derivatives are adequate for the simulation of most maneuvers, including, for example, standard stability and control pulses. There are situations, however, where separate treatment of the translational acceleration derivatives may be necessary for accurate simulation. These situations include short takeoff and landing (STOL) approaches and high angle of attack maneuvers.[13]

Linear Dependence Problems

Several analytical and wind-tunnel techniques are available for predicting translational acceleration derivatives.[1-7] Flight-determined estimates are needed to assist in evaluating these prediction techniques. Currently available flight data analysis programs are algorithmically capable of estimating translational acceleration derivatives. Attempts to estimate translational acceleration derivatives from flight data have been made by using regression[9] and output error[10] maximum likelihood estimation methods. All such attempts have failed. Nguyen[12] used a maximum likelihood method to estimate $\dot{\beta}$ derivatives from noiseless simulated data. Although the technique works well with simulated data, the authors are unaware of any successful application to flight data.

The reasons for the difficulty of estimating translational acceleration derivatives are obvious and well known. Consider the $\dot{\alpha}$ equation:

$$\dot{\alpha} = -\frac{\bar{q}s}{mV} C_L + q + \frac{g}{V}(\cos\theta\cos\varphi\cos\alpha + \sin\theta\sin\alpha)$$
$$-\tan\beta\,(p\cos\alpha + r\sin\alpha) \tag{2}$$

For small perturbations, such as those in standard stability and control pulses, C_L is linearly dependent on α, q, and δ_e. The gravity term (the term containing g/V) is essentially constant, and the last term can be neglected. Thus, $\dot{\alpha}$ is a linear combination of α, q, δ_e, and a bias. The C_m equation has unknown coefficients due to α, q, δ_e and a bias, even without considering $C_{m_{\dot{\alpha}}}$. Thus, the five C_m coefficients are linearly dependent, and they all cannot be estimated. It is only possible to estimate equivalent derivatives ignoring $C_{m_{\dot{\alpha}}}$. This linear dependence problem is inherent in the equations of motion; it is not related to the estimation procedure. Therefore, no estimation program, regardless of its sophistication, can estimate $C_{m_{\dot{\alpha}}}$ from standard aircraft stability and control pulses.

There may be some slight hope of estimating the lateral directional derivatives $C_{l_{\dot\beta}}$ and $C_{n_{\dot\beta}}$ from standard stability and control maneuvers, because the perturbations of the gravity term in the $\dot\beta$ equation give $\dot\beta$ a small component independent of β, p, r, δ_a, and δ_r. There are still problems of near-dependence, however, so noise and modeling errors may preclude the accurate estimation of the $\dot\beta$ derivatives.

Maneuvers That Avoid Linear Dependence

As previously shown, standard stability and control maneuvers contain little information about $\dot\beta$ derivatives, and essentially no information on $\dot\alpha$ derivatives. Rather than attempting derivative estimation from this dearth of information, it seems prudent to consider designing maneuvers that increase the information content of the data. Techniques exist for computing inputs that maximize the information content of the data,[14] but for this nonlinear problem, simple engineering judgment suffices.

The basic problem in estimating $\dot\alpha$ derivatives is the linear dependence of $\dot\alpha$ on the other longitudinal parameters. An open-minded examination of the $\dot\alpha$ equation (Eq. 2) reveals several ways to produce $\dot\alpha$ motion independent of α, q, and δ_e. Large excursions in roll or pitch attitude introduce an independent $\dot\alpha$ component through the gravity term. Large variations in angle of attack must be avoided, however, so that the linear aerodynamic model remains valid. Two maneuvers that meet the criteria of large attitude excursions without large angle-of-attack changes are an aileron roll and a parabola (zoom maneuver). The aileron roll results in much larger changes in the gravity term. For example, in an aileron roll, $\cos\varphi$ ranges from 1 to -1, whereas in a parabola with θ dropping from 45 to -45 deg, $\cos\theta$ ranges from 0.707 to 1. Therefore, attention will be concentrated on the aileron roll.

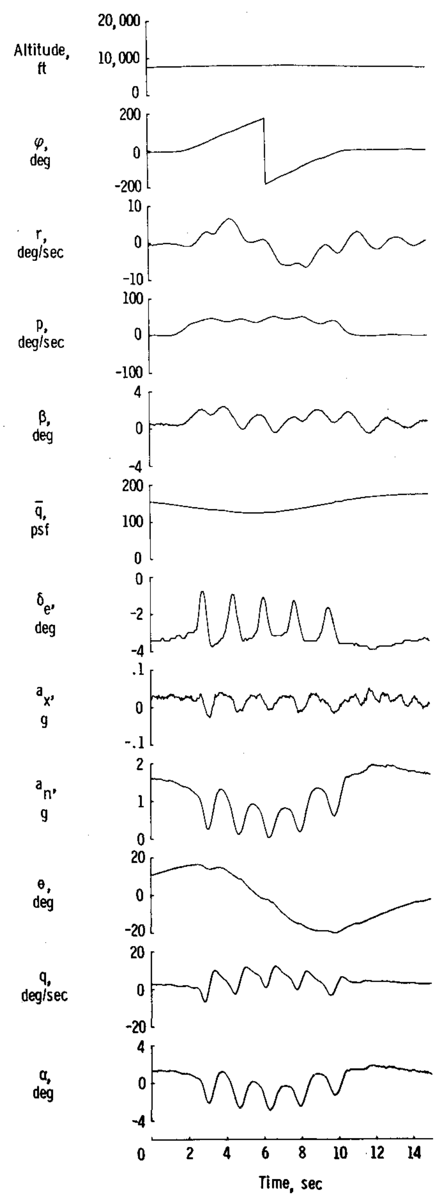

Fig. 3 Time history of aileron roll with elevator pulses.

The aileron roll succeeds in generating a component of $\dot\alpha$ independent of α, q, and δ_e, but it does not excite the longitudinal short-period mode. Thus, longitudinal stability and control derivatives cannot be accurately estimated from an aileron roll alone. To achieve the dual objective of exciting the longitudinal short-period mode and generating independent $\dot\alpha$ motion, a series of elevator pulses is superimposed on the aileron roll. A time history of such a maneuver is shown in Fig. 3.

The aerodynamics of the aileron roll are complicated by roll rate, which is typically 30-60 deg/s. Angle of attack varies along the wingspan, and the flow spirals around the fuselage. Although the effect of roll rate on the longitudinal aerodynamics was believed to be small, a special maneuver was designed to avoid any roll rate effects. This special maneuver is initiated with a standard elevator pulse. The aircraft is then rapidly rolled to an inverted position. An elevator pulse is performed in the inverted position with no roll rate. Finally, the aircraft is rolled back to the upright position. This maneuver is referred to herein as a two-point hesitation roll. Double maneuver analysis[15] is used to analyze the upright and inverted elevator pulses; the rolling portions of the maneuver are not included in the analysis. The double maneuver analysis estimates a single set of derivatives based on the two disjoint sets of data that are included. Since the roll rate is zero during the portions of the maneuver analyzed, the results from the two-point hesitation roll can be compared with the results from the smooth aileron roll to verify that roll rate effects are not important.

Other candidate maneuvers can be designed to obtain independent $\dot\alpha$ motion; for instance, by exploiting the appearance of V and $\bar q$ in the $\dot\alpha$ equation. For this study, the rolling maneuvers hold the most promise, but they are not practical for many large aircraft. The same principles used to design the maneuvers for estimation of $C_{m_{\dot\alpha}}$ can also be used to design maneuvers for estimating $\dot\beta$ derivatives.

Maximum Likelihood Estimation Program

No new development of estimation methodology is necessary to analyze the maneuvers developed for the estimation of translational acceleration derivatives. The analysis reported in this paper was done with the MMLE 3 computer program, an outgrowth of the MMLE program.[15] The MMLE 3 program is a general maximum likelihood estimation program used at the Dryden Flight Research Center. This section briefly describes the features of MMLE 3 that are necessary for the analysis of the newly developed maneuvers, and discusses the implementation of these features in other computer programs.

In general form, the equations of motion are:

$$R(t)\dot x(t) = A(t)x(t) + B(t)u(t) + v(t)$$

$$z(t) = C(t)x(t) + D(t)u(t) + G\eta(t) \qquad (3)$$

The system matrices are time functions because of the variations of $\bar q$, V, θ, and φ during the maneuvers. A time-invariant linear formulation is not adequate. The time-varying formulation may be difficult to handle in some computer programs.

The rolling maneuvers involve significant motion in both the longitudinal and lateral-directional axes. Therefore, the equations of motion used must account for kinematic cross-coupling between the axes. Aerodynamic cross-coupling does not seem to be important. Reference 16 discusses the problems of accounting for cross-coupling terms. Any computer program with a time-varying capability can easily account for the cross-coupling terms.

Naturally, small-angle approximations for θ and φ must be avoided. As in the cross-coupling terms, this does not represent a significant problem if the computer program has time-varying capability.

The derivative $C_{m_{\dot\alpha}}$ is a nondimensional element of the R matrix. Several of the available computer programs do not allow for unknowns in the R matrix or do not even include a fixed R matrix in the formulation. The inclusion of an R matrix with unknowns is a relatively simple modification to most programs.

After the equations of motion are defined, the maximum likelihood estimates are obtained by minimizing the cost functional

$$J(\xi)=\int_0^T [y_\xi(t)-z(t)]^*(GG^*)^{-1}[y_\xi(t)-z(t)]\,\mathrm{d}t \quad (4)$$

where ξ is the vector of unknowns, z is the measured response, and y_ξ is the computed response based on ξ. The MMLE 3 program uses a Newton-Balakrishnan iterative algorithm[15] to perform the minimization.

T-37B Airplane and Instrumentation

In order to test the feasibility of estimating translational acceleration derivatives using the newly developed maneuvers, flight test data were gathered from a T-37B airplane (Fig. 4). The T-37B aircraft is a two-place, straight wing, midtail, jet trainer powered by two J69-T-25 engines.

Reference 17 presents the aircraft's flight-determined stability and control derivatives; translational acceleration derivatives are not included. The aircraft was instrumented with angle-of-attack and sideslip vanes, pitch and roll attitude gyros, three-axis rate gyros and linear accelerometers, total and static pressure transducers, and control position transducers. Fuel weights were read from the cockpit gage and recorded by hand. The data were digitized at 200 samples/s by a nine-bit pulse code modulation (PCM) system and recorded on an onboard tape. Data spikes were removed and the data were passed through a first-order digital low-pass filter with a 20 Hz break frequency. After filtering, the data were thinned to 25 samples/s for analysis.

The roll maneuvers resulted in an altitude gain or loss of as much as 2000 ft in 10 s. Because of the rapid altitude and velocity changes, it was necessary to consider the dynamic characteristics of the total and static pressure transducers. The response of the total pressure transducer was found to be adequate. Step response tests of the static pressure transducer disclosed an approximately exponential response with a 0.4 s time constant. This lag resulted in errors as large as 10% in dynamic pressure. Since the static pressure changes were at low frequencies, a simple time shift of the static pressure data was deemed an adequate correction.

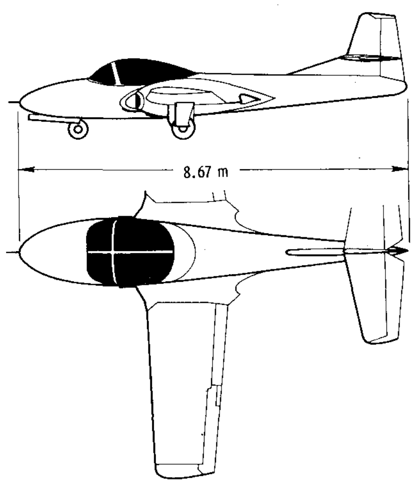

Fig. 4 Two-view drawing of T-37B airplane.

Results and Discussion

Thirteen rolling maneuvers with elevator pulses were performed by the T-37B airplane. Eight were two-point hesitation rolls and five were smooth rolls. These maneuvers were first analyzed neglecting $C_{m_{\dot\alpha}}$. Figure 5 shows a typical fit of one of the smooth rolls. The overall fit is reasonably good, but there are discrepancies in the damping of the measured and computed signals. These discrepancies are most apparent in the overshoots of the a_n and q responses to the

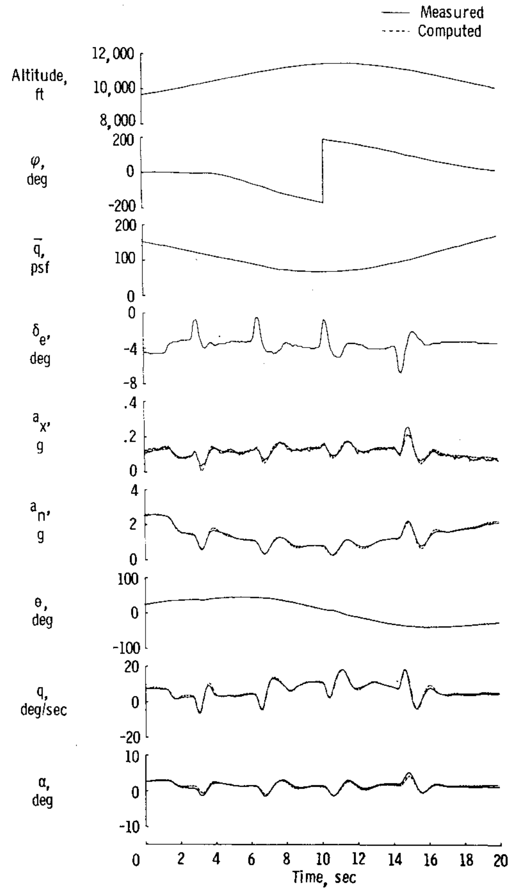

Fig. 5 Fit of smooth roll. C_{m_q} estimated with $C_{m_{\dot\alpha}}$ ignored.

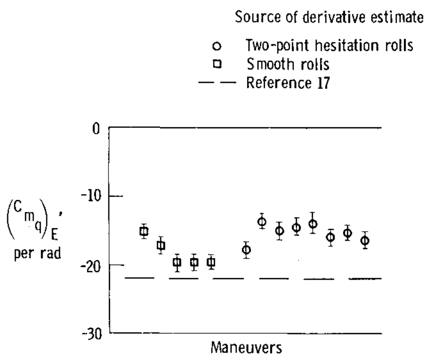

Fig. 6 Flight estimates of equivalent C_{m_q} with $C_{m_{\dot\alpha}}$ ignored.

first and last pulses. The estimates of equivalent C_{m_q} from these maneuvers are given in Fig. 6. The vertical bars in this figure are the Cramer-Rao bounds, which indicate the approximate uncertainty of the estimates. The dashed line is the value of $C_{m_q} + C_{m_{\dot\alpha}}$ obtained from standard stability and control pulses.[17] The values of equivalent C_{m_q} estimated from the smooth roll are about 75% of the values from the standard pulses.

The rolling maneuvers were then analyzed estimating C_{m_q} and $C_{m_{\dot\alpha}}$ independently. Figure 7 shows the fit from this analysis of the same maneuver as in Fig. 5. The fit in Fig. 7 is excellent; in particular, damping is much better matched in Fig. 7 than in Fig. 5. Figure 8 shows the fit of a two-point hesitation roll analyzed neglecting $C_{m_{\dot\alpha}}$. Figure 9 shows the fit of the same maneuver analyzed estimating C_{m_q} and $C_{m_{\dot\alpha}}$ independently. The improvement of Fig. 9 over Fig. 8 is obvious. This satisfies one of the criteria for using a more complicated model; the more complicated model must result in a significant qualitative improvement in the fits. Figure 10 presents the independent estimates of C_{m_q} and $C_{m_{\dot\alpha}}$. The estimates are reasonably consistent from maneuver to maneuver and the Cramer-Rao bounds are good.

The sums of $C_{m_{\dot\alpha}}$ and C_{m_q} from Fig. 10 agree very well with the Ref. 17 estimate of −22. Further, the equivalent C_{m_q} values in Fig. 6 are very close to the C_{m_q} values in Fig. 10. The roll maneuvers remove enough of the relationship between $\dot\alpha$ and q so that Fig. 6 shows relatively pure C_{m_q} estimates rather than $C_{m_q} + C_{m_{\dot\alpha}}$. The estimates of the other stability and control derivatives obtained from the rolling maneuvers are not shown, but they agreed well with the estimates in Ref. 17.

The circles in Fig. 10 indicate the estimates from the two-point hesitation rolls, and the squares indicate smooth rolls. There is no observable trend in the C_{m_q} estimates. The $C_{m_{\dot\alpha}}$ estimates from the smooth rolls are slightly more negative than those from the hesitation rolls. This could be an effect of roll rate on $C_{m_{\dot\alpha}}$, or some other unmodeled effect may cause the differences.

Some rough calculations can be compared with the flight estimates of C_{m_q} and $C_{m_{\dot\alpha}}$. For a straight wing, conventional aircraft like the T-37B, the following easily derived formulae, which are based on tail effects only, provide reasonable first-order estimates:

$$C_{m_q} = -2(57.3)\,\frac{l^2}{c^2}\,C_{L_{\alpha_t}} \tag{5}$$

$$C_{m_{\dot\alpha}} = -2(57.3)\,\frac{l^2}{c^2}\,C_{L_{\alpha_t}}\,\frac{d\epsilon}{d\alpha} \tag{6}$$

We have the approximation[18]

$$\frac{d\epsilon}{d\alpha} = 20C_{L_\alpha}\left(\frac{\lambda^{0.3}}{AR^{0.725}}\right)\left(\frac{3c}{l}\right)^{0.25} = 0.52 \tag{7}$$

Thus, $C_{m_{\dot\alpha}}/C_{m_q}$ should be approximately 0.52, in good agreement with the flight-estimated values, which are about 0.43.

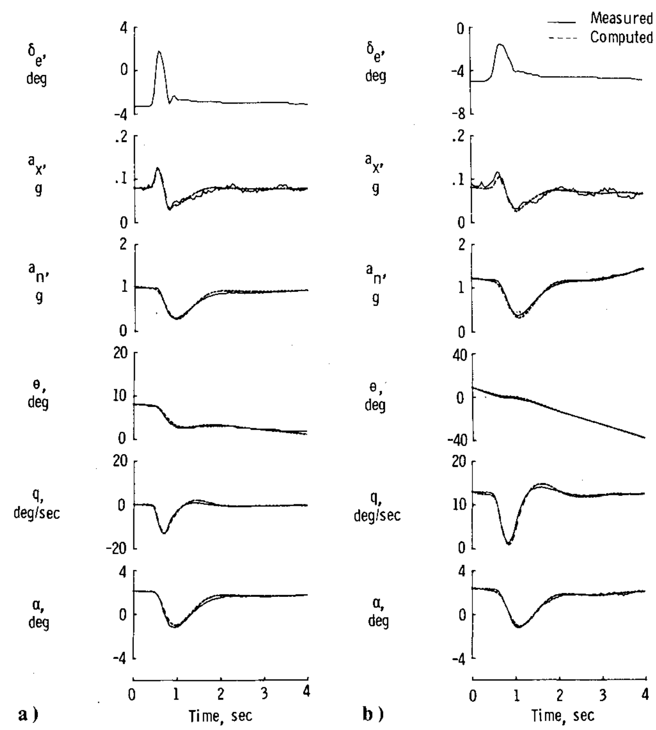

Fig. 8 Fit of two-point hesitation roll. C_{m_q} estimated with $C_{m_{\dot\alpha}}$ ignored. a) Upright elevator pulse; b) inverted elevator pulse.

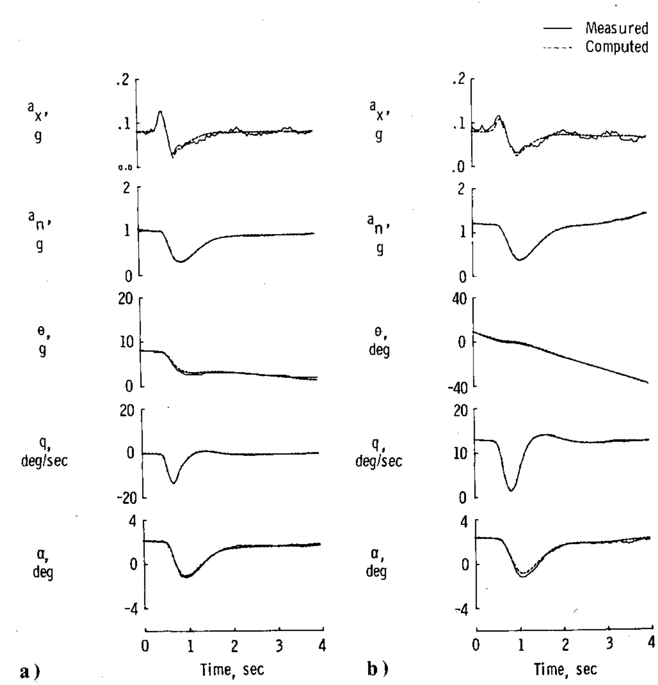

Fig. 9 Fit of two-point hesitation roll. C_{m_q} and $C_{m_{\dot\alpha}}$ estimated independently. a) Upright elevator pulse; b) inverted elevator pulse.

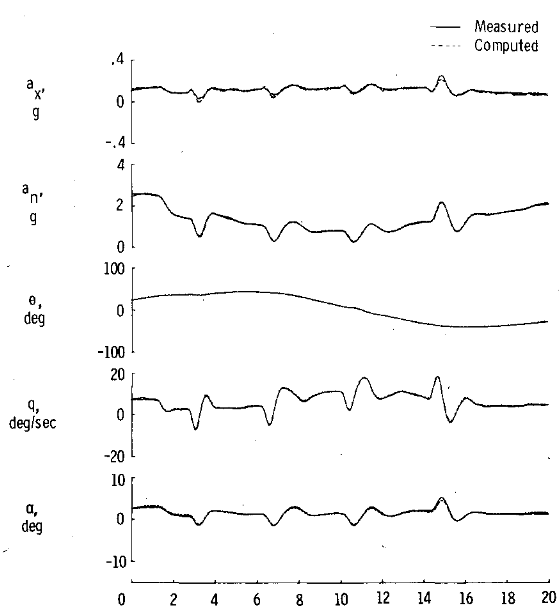

Fig. 7 Fit of smooth roll. C_{m_q} and $C_{m_{\dot\alpha}}$ estimated independently.

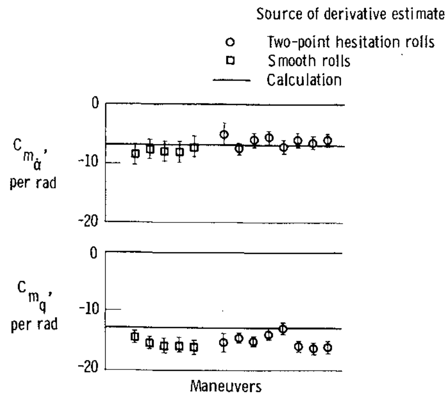

Fig. 10 Independent flight estimates of C_{m_q} and $C_{m_{\dot\alpha}}$.

Another rough calculation from the aspect ratio and taper ratio gives the estimate $C_{L_\alpha} = 0.016$ (nondimensionalized based on the wing reference area). Thus, the predicted C_{m_q} is -13.0 and the predicted $C_{m_{\dot\alpha}}$ is -6.8. The flight-obtained estimates of C_{m_q} and $C_{m_{\dot\alpha}}$, shown in Fig. 10, are in excellent agreement with these calculations. The improvements in the fits, the consistency of the estimates, the good Cramer-Rao bounds, and the agreement with calculations all lend confidence that the maneuvers and estimation techniques used are capable of reliably estimating $C_{m_{\dot\alpha}}$ and C_{m_q} derivatives from flight data.

Conclusions

This paper shows that translational acceleration derivatives can be estimated from flight data if appropriately designed maneuvers are used. No new development of estimation methodology is required to perform the estimation; current maximum likelihood programs are capable of doing the analysis. Consistent, reasonable estimates of $C_{m_{\dot\alpha}}$ with small Cramer-Rao bounds were obtained from roll maneuvers of a T-37B airplane, providing confidence that the maneuvers and estimation techniques used are capable of producing reliable estimates of $C_{m_{\dot\alpha}}$ from flight data.

Appendix—Equations of Motion

The equations of motion used in this paper use small angle approximations for β, but not for α, θ, or φ. Symmetry about the XZ plane is assumed. All angles in these equations are in radians. Except for C_L, all quantities are in body axes. The longitudinal state equations are:

$$\dot\alpha = -\frac{\bar q s}{mV} C_L + q + \frac{g}{V}(\cos\theta\cos\varphi\cos\alpha + \sin\theta\sin\alpha) - \tan\beta(p\cos\alpha + r\sin\alpha)$$

$$\dot q I_Y = \bar q s c C_m + rp(I_Z - I_X) + (r^2 - p^2) I_{XZ}$$

$$\dot\theta = q\cos\varphi - r\sin\varphi$$

The longitudinal observations consist of the states plus the quantities

$$a_n = \frac{\bar q s}{mg} C_N + \frac{XAN}{g}\dot q$$

$$a_x = -\frac{\bar q s}{mg} C_A + \frac{ZAX}{g}\dot q$$

where XAN is the distance of the a_n accelerometer forward of the center of gravity, and ZAX is the distance of the a_x accelerometer below the center of gravity. The expansions of the longitudinal aerodynamic coefficients are:

$$C_N = C_{N_\alpha}\alpha + C_{N_{\delta_e}}\delta_e + C_{N_0}$$

$$C_A = C_{A_\alpha}\alpha + C_{A_{\delta_e}}\delta_e + C_{A_0} - \text{thrust}/(\bar q s)$$

$$C_m = C_{m_\alpha}\alpha + C_{m_q}\frac{qc}{2V} + C_{m_{\dot\alpha}}\frac{\dot\alpha c}{2V} + C_{m_{\delta_e}}\delta_e + C_{m_0}$$

and

$$C_L = C_N\cos\alpha - C_A\sin\alpha$$

The lateral-directional state equations are:

$$\dot\beta = \frac{\bar q s}{mV} C_Y + p\sin\alpha - r\cos\alpha + \frac{g}{V}\cos\theta\sin\varphi$$

$$\dot p I_X - \dot r I_{XZ} = \bar q s b C_l + qr(I_Y - I_Z) + pq I_{XZ}$$

$$\dot r I_Z - \dot p I_{XZ} = \bar q s b C_n + pq(I_X - I_Y) - qr I_{XZ}$$

$$\dot\varphi = p + r\cos\varphi\tan\theta + q\sin\varphi\tan\theta$$

References

[1] Cowley, W.L. and Glauert, H., "The Effect of the Lag of the Downwash on the Longitudinal Stability of an Aeroplane and on the Rotary Derivative M_q," R. & M. No. 718, British A.R.C., 1921.

[2] Jones, R.T. and Fehlner, L.F., "Transient Effects of the Wing Wake on the Horizontal Tail," NACA TN 771, 1940.

[3] Riley, D.R., Bird, J.D., and Fisher, L., "Experimental Determination of the Aerodynamic Derivatives Arising From Acceleration in Sideslip for a Triangular, a Swept, and an Unswept Wing," NACA RM L55A07, 1955.

[4] Orlik-Rückemann, K.J., "Dynamic Stability Testing of Aircraft—Needs Versus Capabilities," *Progress in Aerospace Science*, Vol. 16, 1975, pp. 431-447.

[5] Orlik-Rückemann, K.J., "Recent Advances in Techniques for Dynamic Stability Testing at NAE," DME/NAE Quarterly Bulletin, No. 1976(1), April 1976.

[6] Coe, Jr., P.L., Graham, A.B., and Chambers, J.R., "Summary of Information on Low-Speed Lateral-Directional Derivatives Due to Rate of Change of Sideslip $\dot\beta$," NASA TN D-7972, 1975.

[7] Wells, W.R. and Queijo, M.J., "Simplified Unsteady Aerodynamic Concepts, With Application to Parameter Estimation," *Journal of Aircraft*, Vol. 16, Feb. 1979, pp. 90-94.

[8] Suit, W.T., "Aerodynamic Parameters of the Navion Airplane Extracted From Flight Data," NASA TN D-6643, 1972.

[9] Friedrich, H., "Determination of Stability Derivatives From Flight Test Results by Means of the Regression Analysis," *Methods for Aircraft State and Parameter Identification*, AGARD-CP-172, May 1975, pp. 20-1-20-8.

[10] Iliff, K.W., Maine, R.E., and Shafer, M.F., "Subsonic Stability and Control Derivatives for an Unpowered, Remotely Piloted 3/8-Scale F-15 Airplane Model Obtained From Flight Test," NASA TN D-8136, 1976.

[11] Edward, J.W., "Unsteady Aerodynamic Modeling and Active Aeroelastic Control," SUDAAR 504, Stanford Univ. Dept. of Aeronautics and Astronautics, Feb. 1977.

[12] Nguyen, L.T., "Evaluation of Importance of Lateral Acceleration Derivatives in Extraction of Lateral-Direction Derivatives at High Angles of Attack," NASA TN D-7739, 1974.

[13] Campbell, J.P., Johnson, Jr., J.L., and Hewes, D.E., "Low-Speed Study of the Effects of Frequency on the Stability Derivatives of Wings Oscillating in Yaw With Particular Reference to High Angle-of-Attack Conditions," NACA RM L55H05, 1955.

[14] Gupta, N.K. and Hall, Jr., W.E., "Input Design for Identification of Aircraft Stability and Control Derivatives," NASA CR-2493, 1975.

[15] Maine, R.E. and Iliff, K.W., "A FORTRAN Program for Determining Aircraft Stability and Control Derivatives From Flight Data," NASA TN D-7831, 1975.

[16] Maine, R.E., "Maximum Likelihood Estimation of Aerodynamic Derivatives for an Oblique Wing Aircraft From Flight Data," AIAA Paper 77-1135, Hollywood, Fla., Aug. 1977.

[17] Shafer, M.F., "Stability and Control Derivatives of the T-37B Airplane," NASA TM X-56036, 1975.

[18] Dommash, D.O., Sherby, S.S., and Connolly, T.F., *Airplane Aerodynamics*, Pitman, New York, 1967, p. 444.